A Mapping Technique in Finite Element Method of Magnetic Field Computation for Reduction of Optimization Computation Time

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Abstract — A new efficient mapping technique between the finite element method (FEM) solutions on two sets of meshes, one coarse and one fine, are proposed to reduce the computation time in the study of optimal design problems. In the optimization process of the proposed algorithm, the equation is required to be solved only on the coarse mesh and the more accurate solution on the fine mesh is obtained by mapping. The proposed mapping technique is verified by solving TEAM workshop problem No. 25 and the computation time of the proposed method is only 35.5% of that required by the general methods.

I. Introduction

Finite element method (FEM) is widely used in electromagnetic device analysis and design [1]. However, it takes one or two hours for one design using FEM and most optimization methods have to execute the object function many thousands times before approaching the optimal solution [2]. In some cases, the computation time of FEM can be reduced by reducing of the number of elements in the mesh at the expense of accuracy.

In optimization process, a change in the shape of the device being optimized requires different FEM solutions. It is also noted that all the FEM solutions in the optimization process have some common characteristics because it shares similar computation domain, the same boundary condition, the same distribution of materials, even the same excitations in most cases. Moreover, the FEM solution consists of two parts: one part is the main change, which is the low frequency part in the FEM solution, and another part is the vibrations on the previous part, which is the high frequency part in the FEM solution. The low frequency part is very dependent on the optimization parameters, while the high frequency attached on the low frequency part is not. The low frequency part of the solution may be obtained with large intervals on the computational domain and the high frequency part should be captured on small intervals.

In the proposed method, two set of meshes υ_H and υ_h (with mesh size $h << H$) are constructed for the FEM discretization at each optimization step. Usually the fine mesh υ_h can be chosen as the adaptive refinement on the coarse mesh υ_H. The difference in the solutions between the fine and coarse meshes are the high frequency part. Because the high frequency part is less dependent on optimization parameters, a mapping can be constructed to link the high frequency part and the low frequency part which is the solution on the coarse mesh. The mapping can be setup by solutions of tens of FEM computation on the coarse and fine meshes with the optimization parameters randomly chosen. The solution obtained after mapping is expected to be reasonably accurate as it is the FEM solution on the fine mesh. Since $\dim(\upsilon_H) << \dim(\upsilon_h)$, the effort for solving the problem on the coarse mesh is relatively simple. In other words, the mapping upholds the accuracy of the FEM solution and at the same time reduces the computation time.

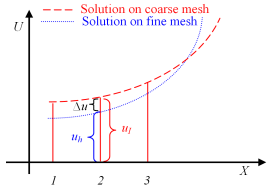


Fig.1. Difference between the solution on the coarse mesh and fine mesh.

II. Mapping Method

The mapping method can be illustrated by a simple example as shown in Fig. 1. The dotted line represents the solution on the coarse mesh and the solid line represents the solution on the fine mesh. Node 2 is in the fine mesh and the adjacent nodes of node 2 on the coarse mesh are nodes 1 and 3. The solution on the fine mesh on node 2 is u_h, and the interpolation of the solution on the coarse mesh to node 2 is u_I. The difference between u_h and u_I is $\Delta u = u_h - u_I$. Δu can be represented by the linear combination of u_I, u_1, u_2, u_1', u_2' and the coordinate of node 2, x. $\Delta u = au_i + b(u_1+u_2) + c(u_1' + u_2') + dx$, where u_1' *and* u_2' are the derivative of the solution on u_1 and u_2, respectively. The coefficient a, b, c, d can be obtained by the least square fitting on solution of the *N-th* time on the coarse mesh and fine mesh:

$$min\sum (\Delta u_i - au_{Ii} - b(u_{1i}+u_{2i}) - c(u_{1i}' + u_{2i}') - dx)^2 \qquad (1)$$

where Δu_i, and $u_{Ii}, u_{1i}, u_{2i}, u_{1i}', u_{2i}'$ are the i-th time solution. Once the coefficients are determined, the solution on the fine mesh can be obtained from the solution on the coarse mesh $u_h = u_I + \Delta u = u_I + au_i + b(u_1+u_2) + c(u_1' + u_2') + dx$.

The representation of Δu includes the consideration of the solution on the nodes itself, the solution on its adjacent nodes, the derivative on the element on the coarse mesh and the position of the nodes together, which, indeed, represents the high frequency part of the solution.

The above mapping method can be extended directly to 2-dimension (2D) and three-dimension (3D) problems. For each node on the fine mesh, there exists one element in the coarse mesh which contains it. The nodes on that element can be chosen as the adjacent of the node on the fine mesh.

In the mapping method, not all of the nodes on the fine mesh requires mapping. Only the nodes which are related with the objective function calculation require mapping. This will reduce the number of the unknown coefficients.

The mapping algorithm requires a comparison between the N solutions of the coarse mesh and fine mesh. The i-th time can be stated as:

1. Determine the shape of the devices by selecting the optimization parameters randomly; generate both coarse and fine meshes on the computation domain.
2. Solve the equation on the coarse and fine meshes using FEM.
3. Determine the set of nodes on the fine mesh $\{u_h^1, u_h^2, u_h^3, \dots, u_h^n\}$, which is related with the objective function calculation and at the same time obtain the adjacent nodes for each nodes in $\{u_h^1, u_h^2, u_h^3, \dots, u_h^n\}$.
4. Interpolate the solution on the coarse mesh to the nodes $\{u_h^1, u_h^2, u_h^3, u_h^n\}$ by interpolation and obtain Δu_i.
5. Store the Δu_i on the nodes $\{u_h^1, u_h^2, u_h^3, \dots, u_h^n\}$ for the calculation of the coefficients *a, b, c, d*.

After *N* times of FEM computation, the coefficient *a, b, c, d* can be calculated by the stored Δu_i to realise the mapping.

III. Interpolation from Coarse Mesh to Fine Mesh

Accurate interpolation of the solution from the coarse mesh to the fine mesh is an important procedure of the proposed method. The proposed interpolation is based on the coarse FEM space v_H. Given the basic function on v_H is $\{\psi_i\}_{i=1}^n$ and the solution is $\{u_i\}_{i=1}^n$, then if node $v(x,y,z)$ is in the fine mesh, the interpolation value on node *v* is:

$$u(v) = \sum_{i=1}^{n} u_i \psi_i\left(\xi(x,y,z), \eta(x,y,z), \zeta(x,y,z)\right) \qquad (2)$$

where (ξ, η, ζ) is the reference coordinates. To determine the reference coordinates of node *v*, the coarse mesh element which contains it must be found according to its global coordinates (*x, y, z*). There are two procedures to find the element. Firstly one must determine a set of elements that probably contain the point according to the global coordinates of the node in the element. The second step is to solve the following nonlinear algebraic transformation equation by Newton iteration method on each element in the set:

$$x = f(\xi, \eta, \zeta),\ y = g(\xi, \eta, \zeta),\ z = h(\xi, \eta, \zeta) \qquad (3)$$

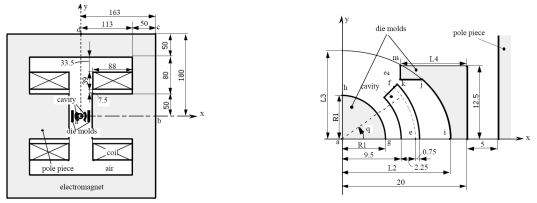
where *f, g, h* are shape functions on the element.

The reference coordinates (ξ, η, ζ) can be obtained in the set of elements. If (ξ, η, ζ) is in the defined range, the element that contains the node is found and the interpolation values on the node can be calculated by (2).

IV. Numeric Example

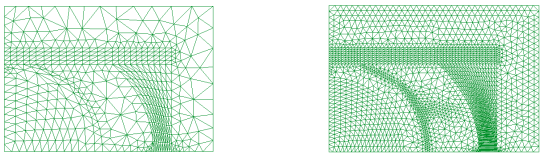
The proposed method is applied to TEAM workshop problem No. 25 [4]. The goal of this problem is to optimize the shape of a die mold to obtain the best performance of permanent magnets. Fig. 2(a) shows the model of the die mold with the electromagnet for the orientation of the magnetic axis of the magnetic powder. The die mold is described by an internal circle of radius R_1 and by an external ellipse represented by L_2, L_3 and L_4, as shown in Fig. 2(b). The objective function *W* for the optimization problem is given by: $W = \sum_{i=1}^{n}\left[\left(B_{xp_i} - B_{xo_i}\right)^2 + \left(B_{yp_i} - B_{yo_i}\right)^2\right]$

where; *n* is the number of specified points; B_{xpi} and B_{ypi} are the computed values along the line e-f; B_{xi} and B_{yo} are specified as $B_{xo}=1.5\cos(\theta)$, $B_{yo}=1.5\sin(\theta)$ (T).



(a) (b)

Fig. 2. (a) Die mold with electromagnet. (b) Optimization parameters



(a) (b)

Fig 3. (a) Coarse mesh (b) Fine mesh

Fig. 3 shows the coarse mesh with 519 nodes and the fine mesh with 4216 nodes on the computation domain of one design. Using particle swarm optimization method, the swarm size is set as 20 and the stop criteria is defined by two performance parameters $\Delta B_{\max} = \max((B_p - B_o)/B_o) < 3\%$ and $\Delta\theta_{\max} = \max(|\theta_{Bp} - \theta_{Bo}|) < 2^o$. In the mapping construction process, it takes about 247 seconds to do 400 times of FEM computation on the coarse and fine meshes (including nonlinear iteration and derivative calculation). As is shown in Table I, it takes about 144 minutes to do 7860 times FEM computation by the proposed method, while it takes about 418 minutes to do 5880 times FEM computation on the fine mesh using the conventional method. For 3D problems which requires much more time to do FEM computation, the improvement by the proposed method will be more significant.

TABLE I
COMPARISON OF THE COMPUTATION TIME

Method	Calculation	FEM Times	Time (s)	Total time (s)
Proposed method	Mapping	400	247	8893
	Coarse mesh	7860	8646	
General method	Fine mesh	5880	25082	25082

V. References

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